

La production

Fonction de production:

$$q = f(K, L)$$

Exemple: Cobb-Douglas: $q = AK^\alpha L^\beta$

1) Principe de non gaspillage

2) Facteurs fixes et variables (court terme et long terme)

3) Progrès technique (différents types):

$$q_t = e^{at} f(K_t, L_t) ; q_t = f(K_t, e^{bt} L_t)$$

3) Rendement d'échelle:

$$A(\gamma K)^\alpha (\gamma L)^\beta = (\gamma)^{\alpha+\beta} AK^\alpha L^\beta = (\gamma)^s q$$

$s = \alpha + \beta =$ rendement d'échelle

$s > 1$ rendement d'échelle croissant

$s = 1$ rendement d'échelle constant

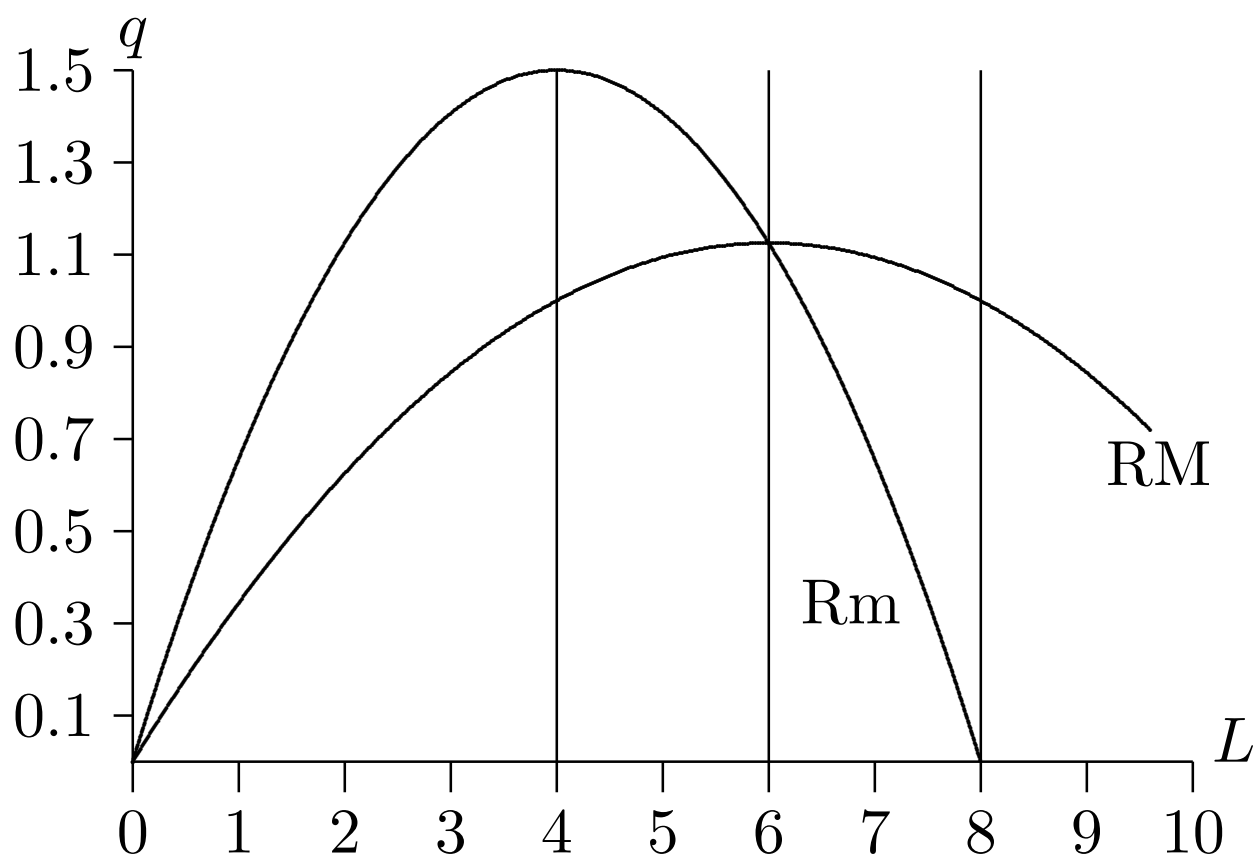
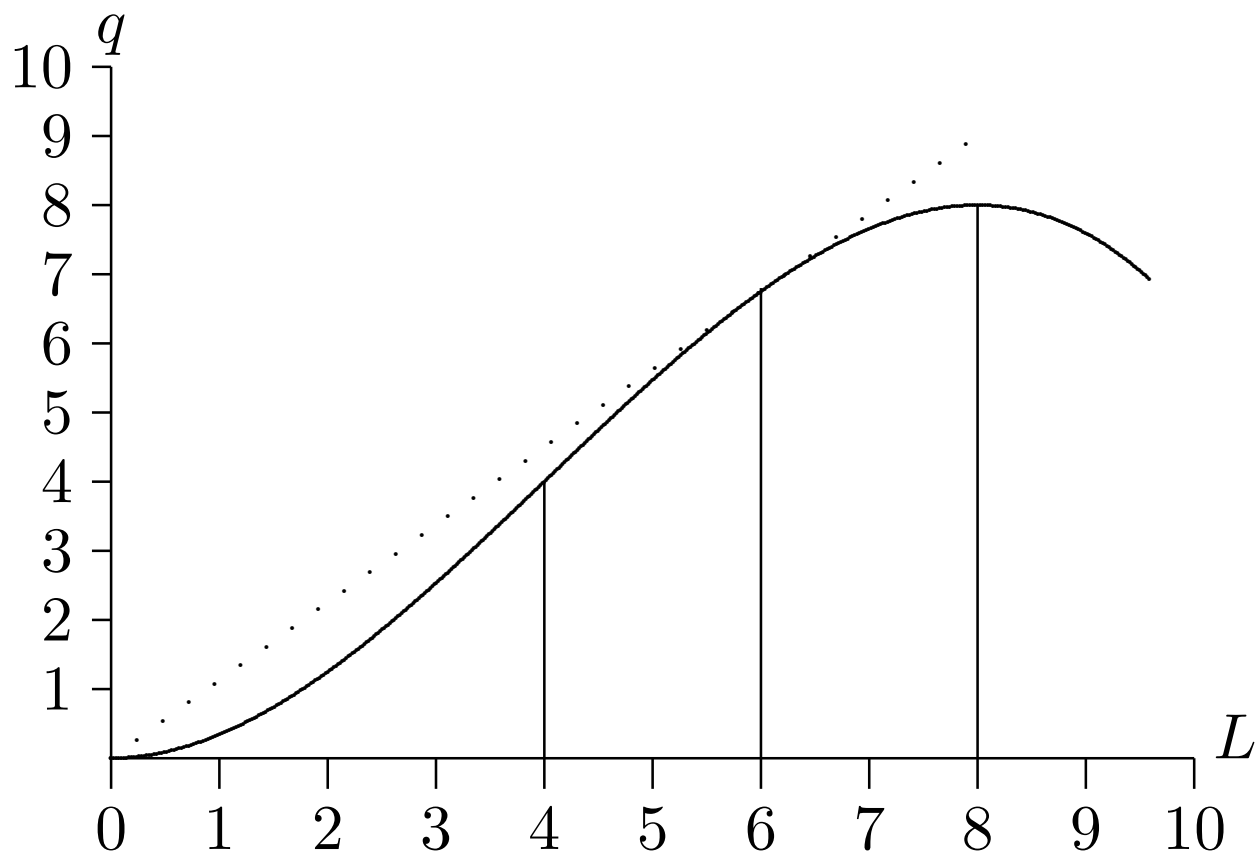
$s < 1$ rendement d'échelle décroissant

4) Rendement marginal: loi des rendements marginaux décroissants

$$\frac{\partial q}{\partial K} = f_K > 0 ; \frac{\partial^2 q}{\partial K^2} = f_{KK} < 0$$

$$\frac{\partial q}{\partial L} = f_L > 0 ; \frac{\partial^2 q}{\partial L^2} = f_{LL} < 0$$

Productivité totale, moyenne et marginale



Productivité totale, moyenne et marginale

Fonction de production:

$$q = A(3K^2L^2 - \frac{1}{8}K^3L^3)$$

Productivité moyenne et marginale du travail lorsque $K = 2$:

$$\frac{q}{L} = A(12L - L^2)$$

$$\frac{\partial q}{\partial L} = A(24L - 3L^2)$$

Relation entre valeur totale (T) , moyenne (M) et marginale (m):

$$T = M \times x \quad ; \quad M = \frac{T}{x}$$

$$\frac{\partial T}{\partial x} = m = \frac{\partial M}{\partial x} x + M$$

$$\frac{\partial M}{\partial x} = \frac{(m-M)}{x}$$

La moyenne (M) augmente si $m > M$

La moyenne (M) diminue si $m < M$

La moyenne (M) a une valeur stationnaire si $m = M$

Dans le graphique ci-joint on a pris $A = \frac{1}{32}$

Types de fonction de production

1) Cobb-Douglas: $q = AK^\alpha L^\beta$

$$\frac{\partial q}{\partial K} = \alpha AK^{\alpha-1} L^\beta = \alpha \frac{q}{K}$$

$$\frac{\partial^2 q}{\partial K^2} = \alpha(\alpha - 1)AK^{\alpha-2} L^\beta < 0 \text{ si } 0 < \alpha < 1$$

$$\frac{\partial q}{\partial L} = \beta AK^\alpha L^{\beta-1} = \beta \frac{q}{L}$$

$$\frac{\partial^2 q}{\partial L^2} = \beta(\beta - 1)AK^\alpha L^{\beta-2} < 0 \text{ si } 0 < \beta < 1$$

2) Leontief: $q = \min(\frac{K}{a}, \frac{L}{b})$

3) CES: $q = A[\alpha K^{-\rho} + (1 - \alpha)L^{-\rho}]^{\frac{-s}{\rho}}$

$\sigma = \frac{1}{1+\rho}$ $s =$ rendement d'échelle

Si $\rho = 0$ $\sigma = 1$ on obtient Cobb-Douglas

Si $\rho = \infty$ $\sigma = 0$ on obtient Leontief

Si $\rho = -1$ $\sigma = \infty$ on obtient une isoquante qui est une droite

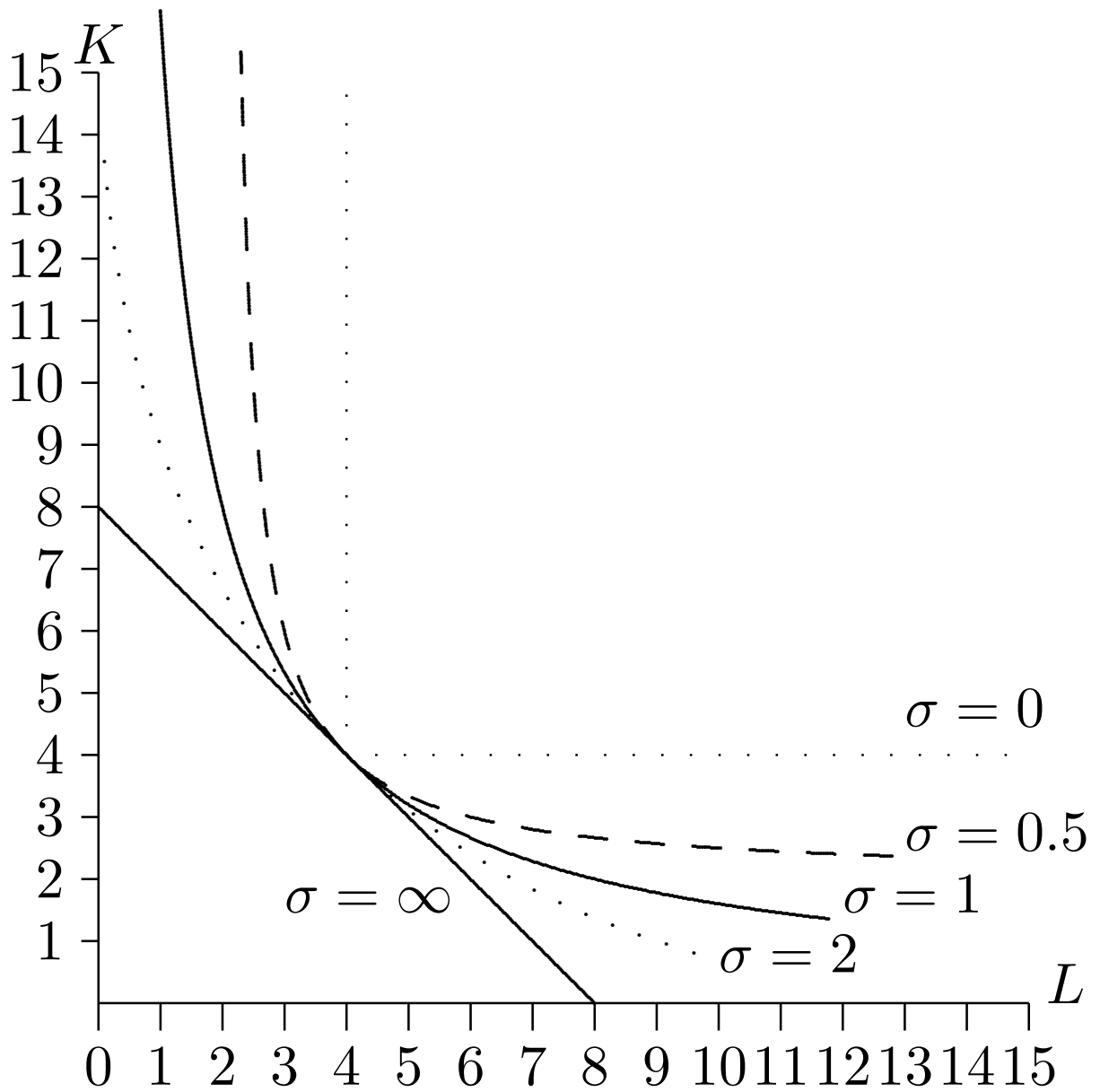
Si $\rho < -1$ $\sigma < 0$ on obtient une isoquante concave (CET)

4) Programmation linéaire:

$$q^1 = \min(\frac{K_1}{a_1}, \frac{L_1}{b_1}) ; q^2 = \min(\frac{K_2}{a_2}, \frac{L_2}{b_2})$$

$$q = q^1 + q^2$$

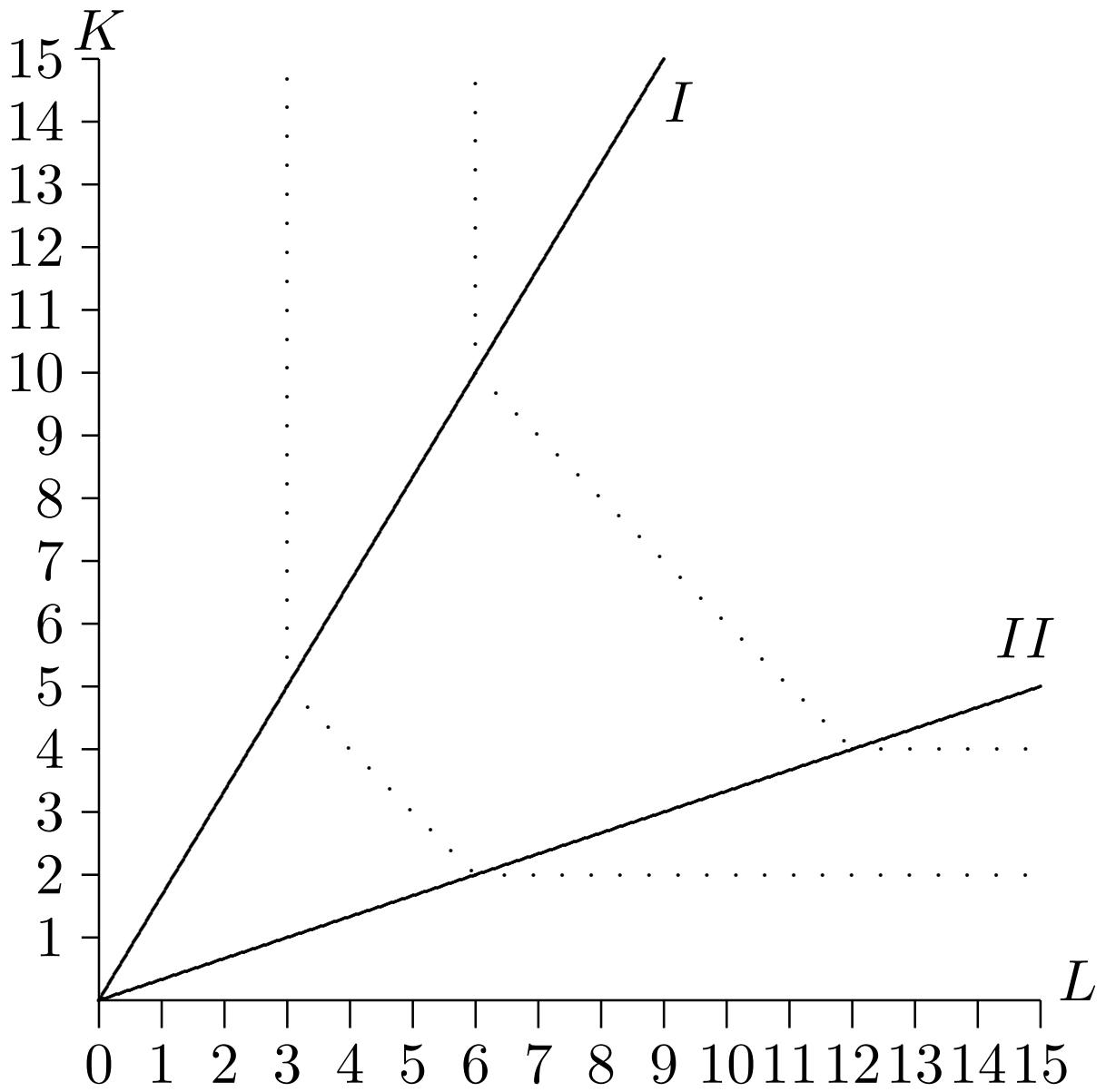
Fonction de production CES



$$q = [aK^{-\rho} + (1 - a)L^{-\rho}]^{-1/\rho} \quad a = 0.5$$

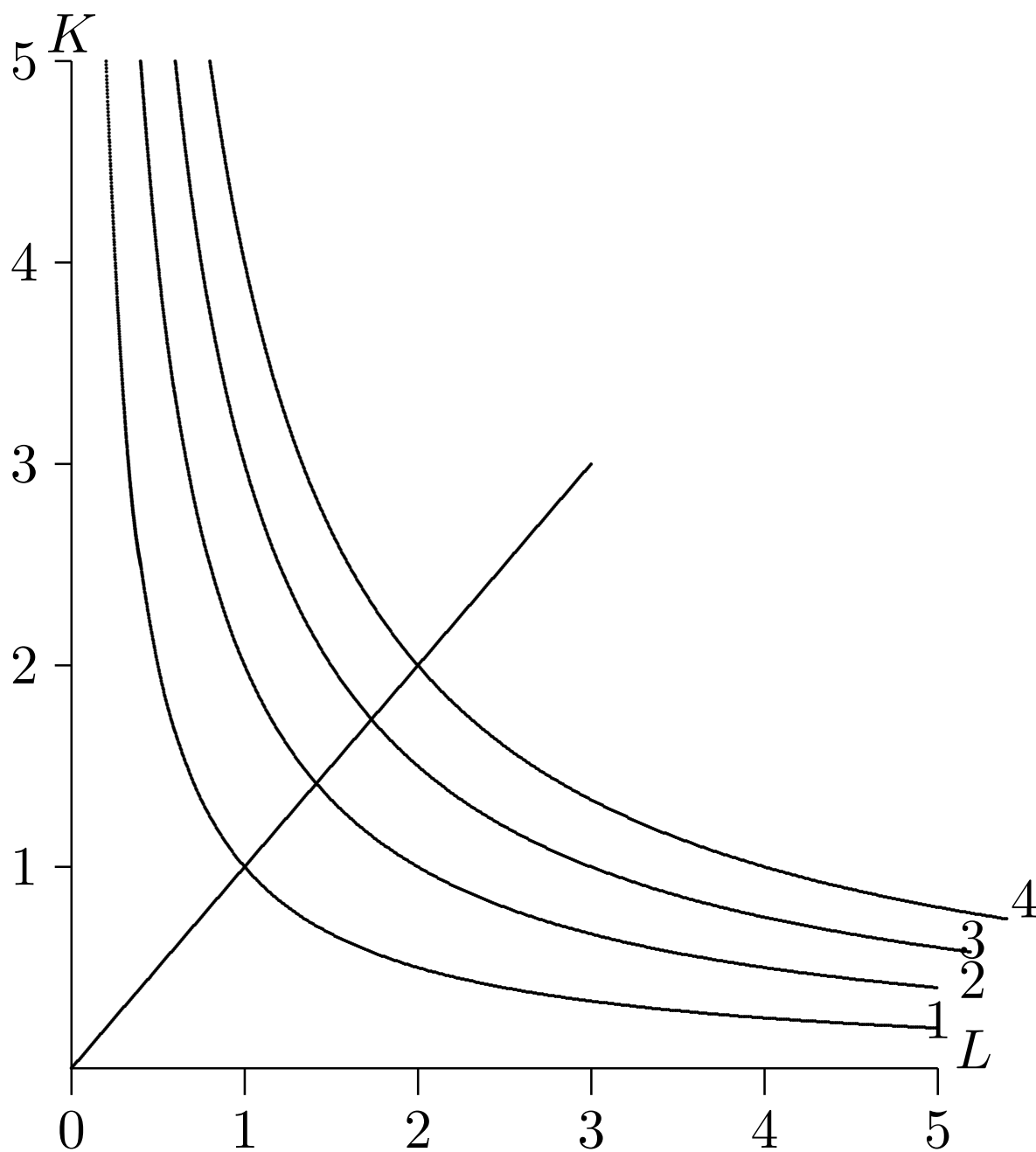
$$\sigma = 1/(1 + \rho)$$

Substitution discontinue



$$q^I = \min\left(\frac{K}{5}, \frac{L}{3}\right) \quad ; \quad q^{II} = \min\left(\frac{K}{2}, \frac{L}{6}\right)$$

Rendement d'échelle et isoquantes



$$q = KL \quad ; \quad s = 2$$

Choix des facteurs

$$\min C = p_K K + p_L L \quad \text{S.C.} \quad q = f(K, L)$$

$$\mathcal{L} = p_K K + p_L L + \lambda[q - f(K, L)]$$

$$\begin{cases} \frac{\partial \mathcal{L}}{\partial K} = p_K - \lambda \frac{\partial q}{\partial K} = 0 & (a) \\ \frac{\partial \mathcal{L}}{\partial L} = p_L - \lambda \frac{\partial q}{\partial L} = 0 & (b) \\ \frac{\partial \mathcal{L}}{\partial \lambda} = q - f(K, L) = 0 & (c) \end{cases}$$

En résolvant on obtient:

$$\frac{p_K}{f_K} = \frac{p_L}{f_L} = \lambda ; \quad \frac{f_K}{p_K} = \frac{f_L}{p_L} ; \quad \frac{p_L}{p_K} = \frac{f_L}{f_K}$$

$$\lambda = \frac{\partial \mathcal{L}}{\partial q} = Cm$$

$$K = \phi_1(p_K, p_L, q) ; \quad L = \phi_2(p_K, p_L, q)$$

Exemples: (1) $q = \sqrt{KL}$

$$K = q \sqrt{\frac{p_L}{p_K}} ; \quad L = q \sqrt{\frac{p_K}{p_L}} ; \quad C = 2q \sqrt{p_K p_L}$$

$$(2) \quad q = AK^\alpha L^\beta$$

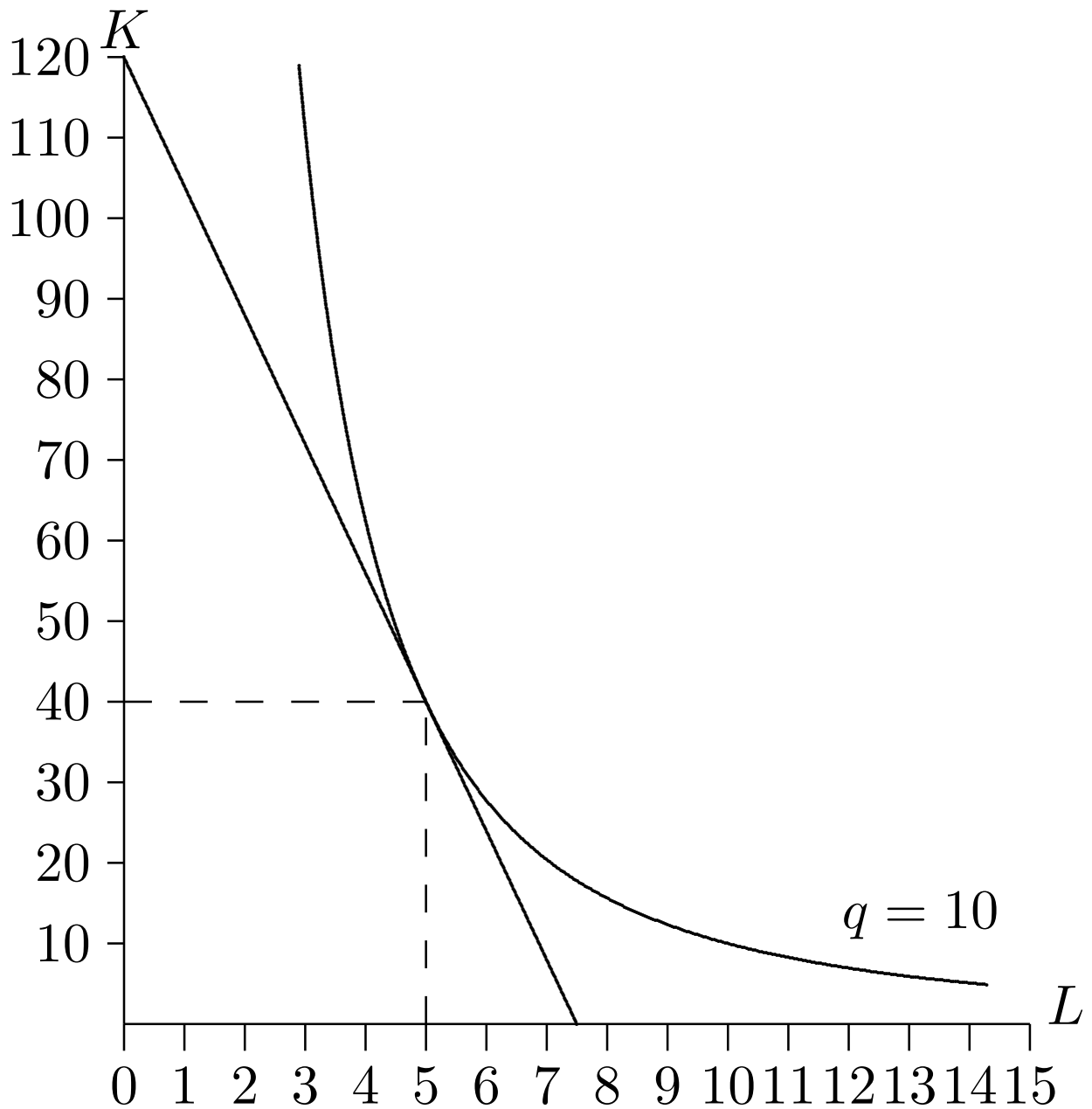
$$K = q^{1/s} A^{-1/s} \left(\frac{\alpha p_L}{\beta p_K} \right)^{\beta/s}$$

$$L = q^{1/s} A^{-1/s} \left(\frac{\beta p_K}{\alpha p_L} \right)^{\alpha/s}$$

$$C = q^{1/s} s A^{-1/s} \left(\frac{\beta p_K}{\alpha p_L} \right)^{\alpha/s} \frac{p_L}{\beta}$$

$$s = \alpha + \beta$$

Choix des facteurs

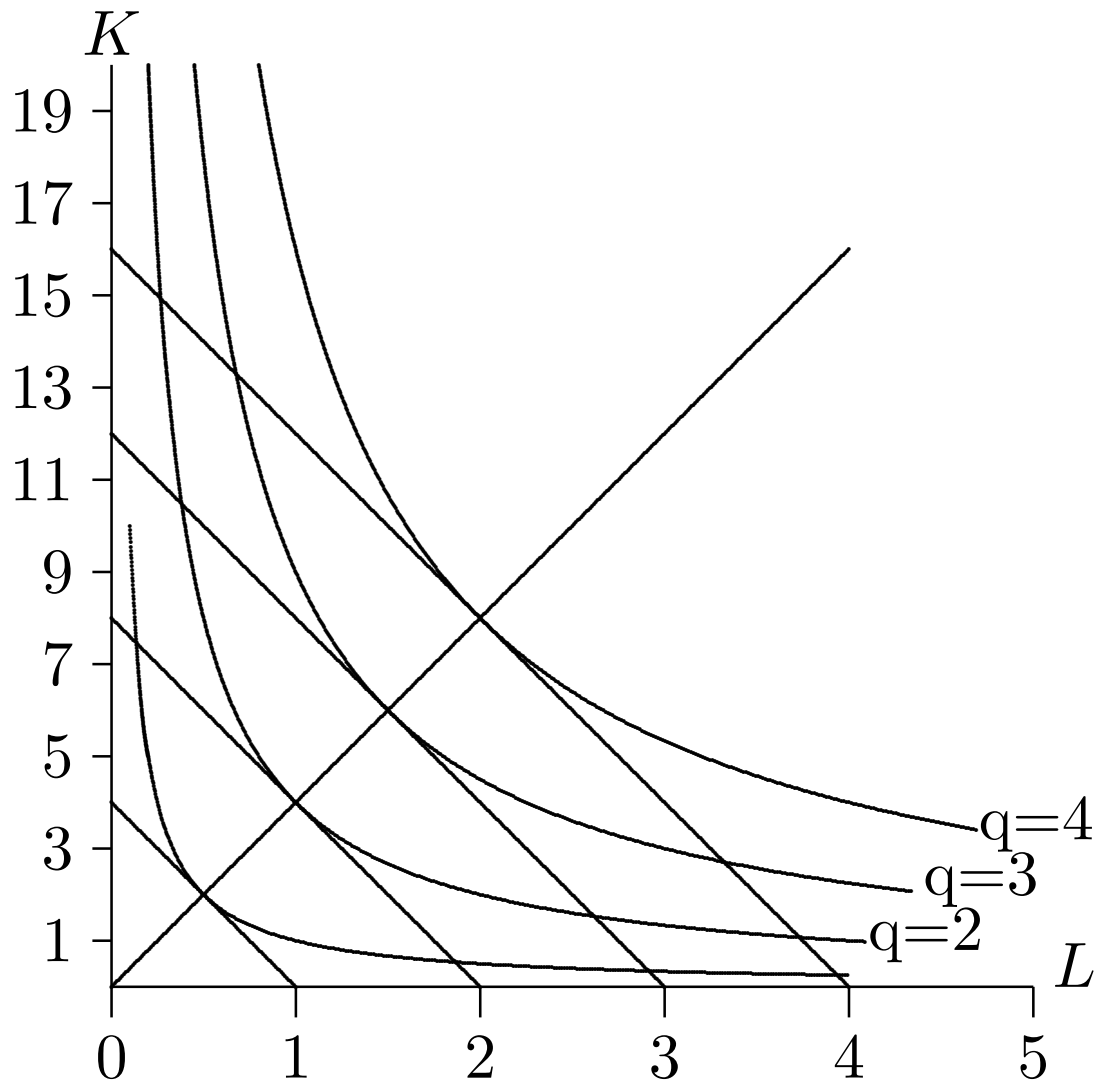


$$q = K^{1/3} L^{2/3} ; p_K = 1 ; p_L = 16$$

$$\text{pente de l'isocoût} = \frac{p_L}{p_K} = \frac{16}{1}$$

$$\text{pente de l'isoquante} = \text{TST} = \frac{f_L}{f_K} = \frac{2K}{L} = 16$$

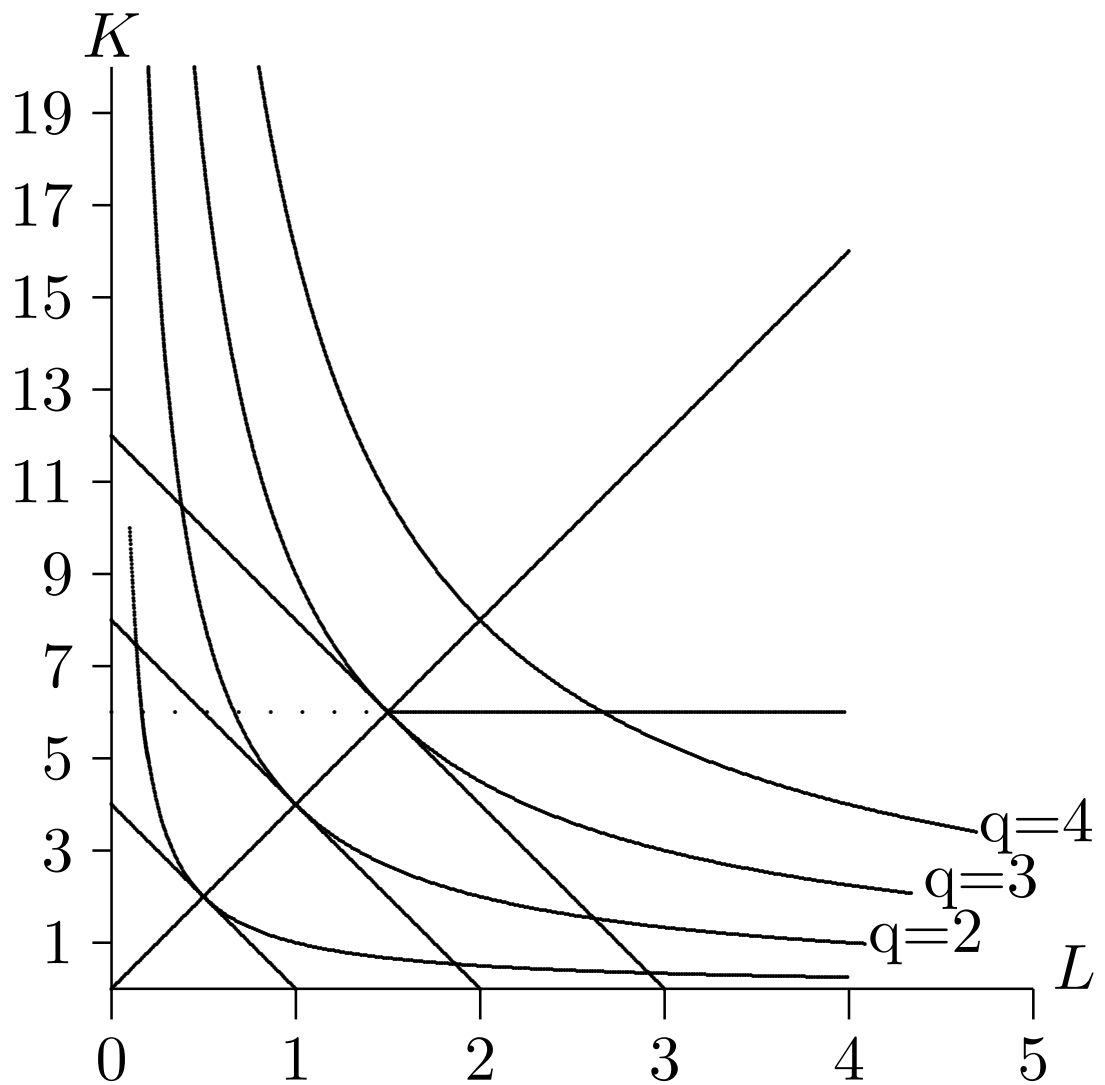
Chemin d'expansion



$$q = \sqrt{KL} \quad p_K = 1 \quad p_L = 4$$

$$K = 2q \quad ; \quad L = 0.5q$$

Chemin d'expansion à court terme



$$q = \sqrt{KL} \quad p_K = 1 \quad p_L = 4$$

$$K^o = 6$$

Coût total, moyen et marginal

Fonction de coût:

$$C = c_o + aq + bq^2 + dq^3$$

avec $c_o > 0$ (coûts fixes)

$$a, d > 0 ; b < 0 ; b^2 < 3ad$$

Coût moyen et marginal:

$$CM = \frac{c_o}{q} + a + bq + dq^2$$

$$Cm = a + 2bq + 3dq^2$$

Le coût moyen variable est:

$$CMV = a + bq + dq^2$$

Le profit de l'entreprise est:

$$\Pi = R - C$$

La maximisation du profit donne:

$$\frac{d\Pi}{dq} = Rm - Cm = 0$$

Condition de premier ordre:

$$Rm = Cm$$

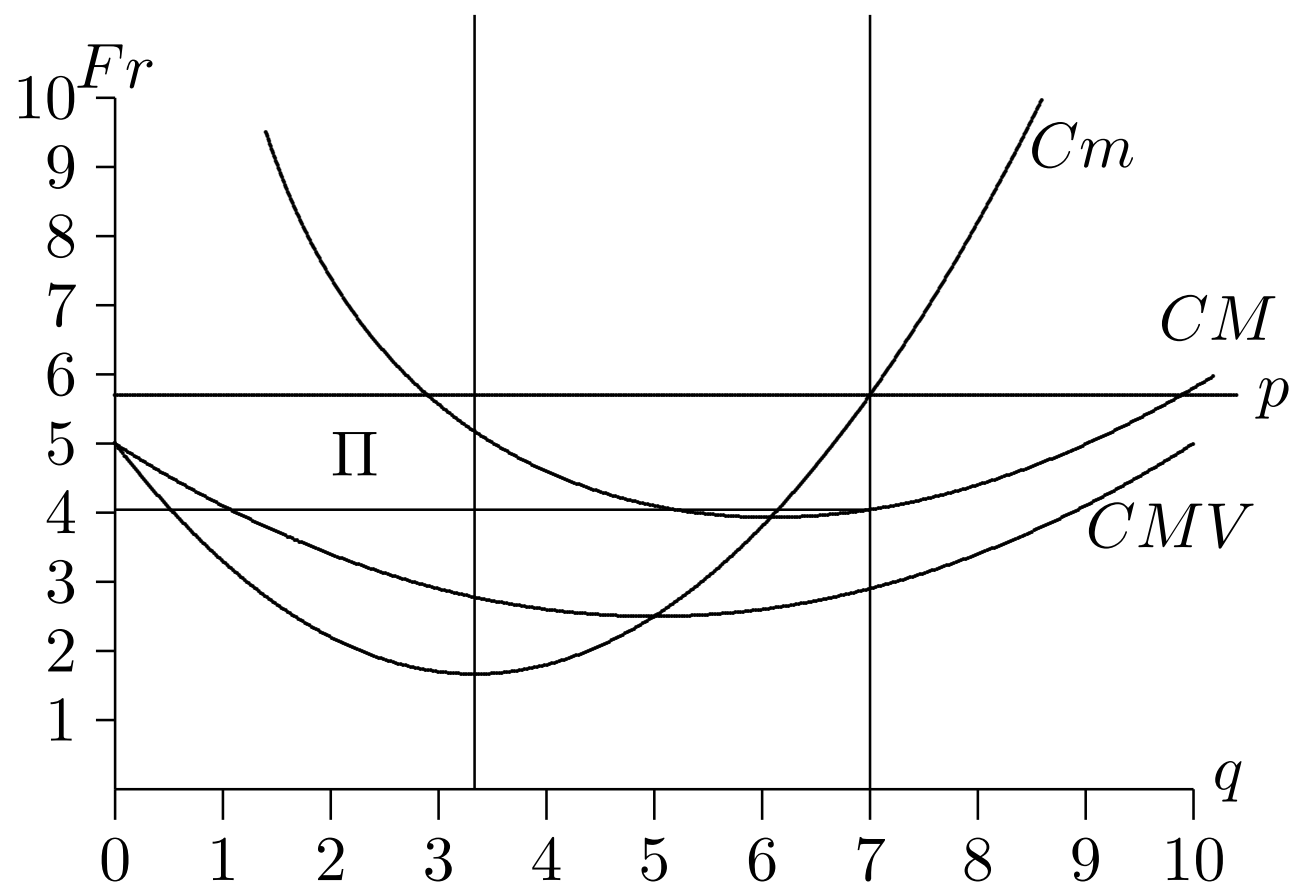
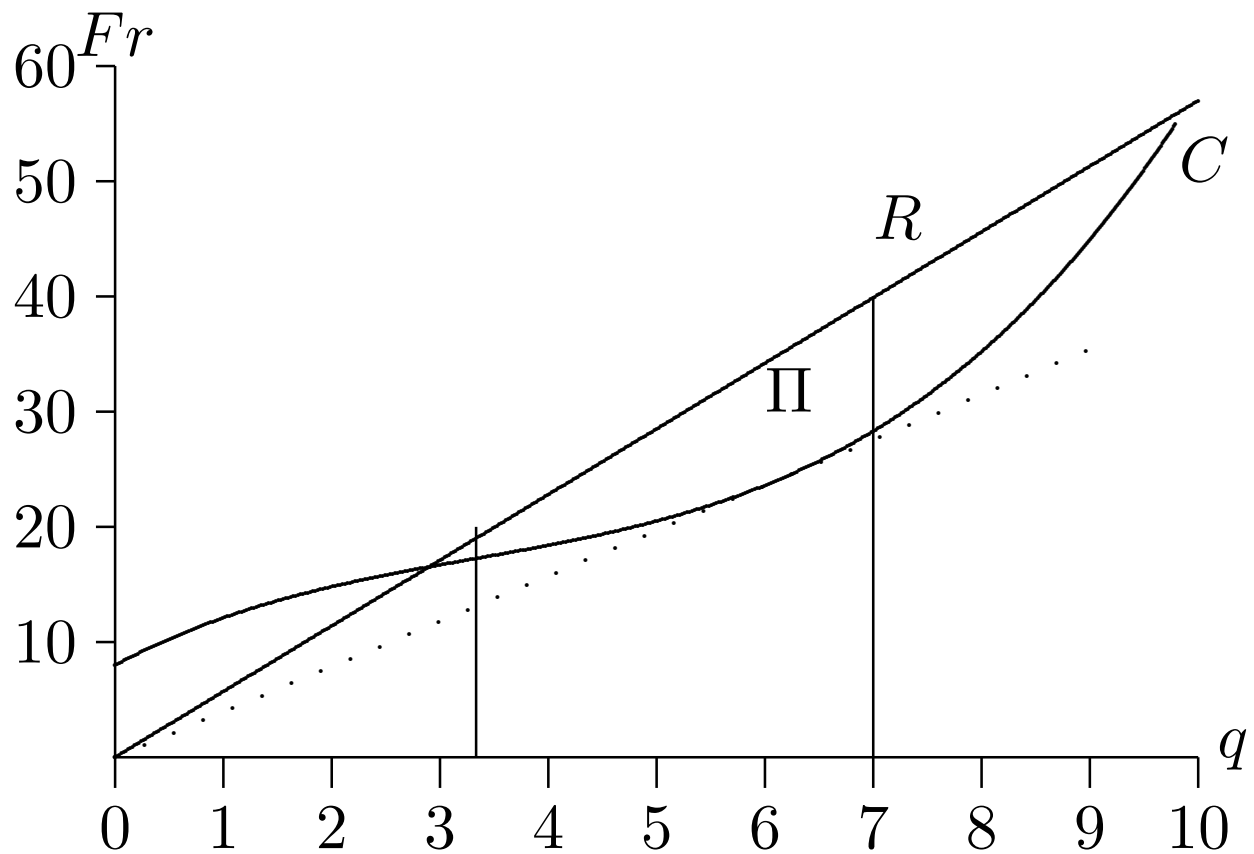
Condition de deuxième ordre:

$$\frac{d^2\Pi}{dq^2} = \frac{dRm}{dq} - \frac{dCm}{dq} < 0$$

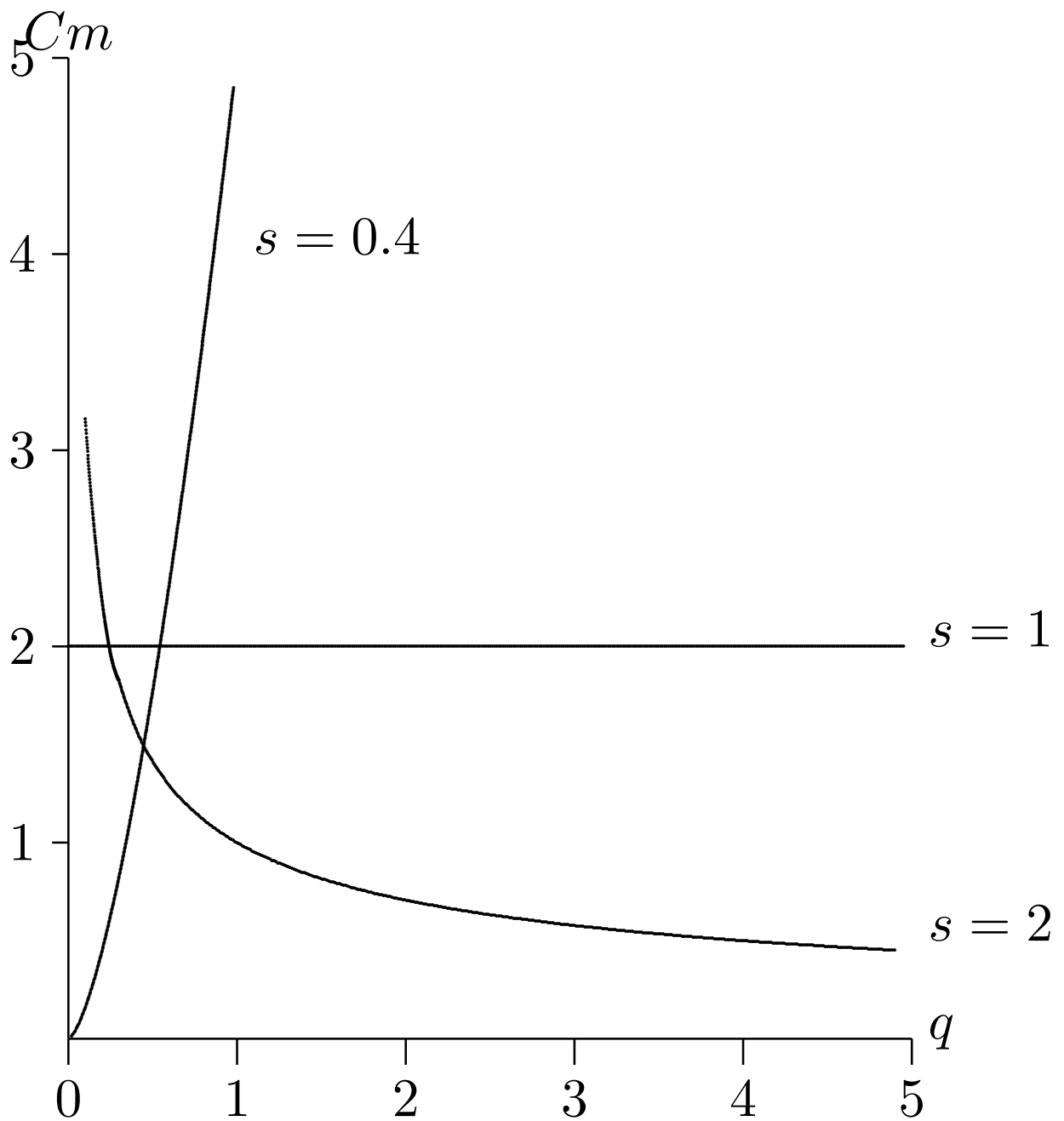
Dans le graphique ci-joint on a pris:

$$C = 8 + 5q - q^2 + 0.1q^3 \quad \text{et} \quad p = 5.7$$

Coût total, moyen et marginal



Rendement d'échelle et coût marginal



$$q = AK^\alpha L^\beta$$

$$Cm = q^{\frac{1-s}{s}} A^{-1/s} \left(\frac{\beta p_K}{\alpha p_L} \right)^{\alpha/s} \frac{p_L}{\beta}$$

$$s = \alpha + \beta$$

Objectifs de l'entreprise

$$p_1 = 24 - q_1 \quad ; \quad p_2 = 16 - 0.5q_2$$

$$C = 20 + 4(q_1 + q_2)$$

1) Maximisation du profit:

$$\Pi = 24q_1 - q_1^2 + 16q_2 - 0.5q_2^2 - 20 - 4(q_1 + q_2)$$

$$\frac{\partial \Pi}{\partial q_1} = 24 - 2q_1 - 4 = 0 \quad q_1 = 10 \quad ; \quad p_1 = 14$$

$$\frac{\partial \Pi}{\partial q_2} = 16 - q_2 - 4 = 0 \quad q_2 = 12 \quad ; \quad p_2 = 10$$

$$R = 260 \quad ; \quad \Pi = 152$$

2) Maximisation du chiffre d'affaires:

$$\Pi = 24q_1 - q_1^2 + 16q_2 - 0.5q_2^2$$

$$\frac{\partial \Pi}{\partial q_1} = 24 - 2q_1 = 0 \quad q_1 = 12 \quad ; \quad p_1 = 12$$

$$\frac{\partial \Pi}{\partial q_2} = 16 - q_2 = 0 \quad q_2 = 16 \quad ; \quad p_2 = 8$$

$$R = 272 \quad ; \quad \Pi = 140$$

3) Maximisation du chiffre d'affaires avec contrainte ($\Pi = 149$):

$$L = 24q_1 - q_1^2 + 16q_2 - 0.5q_2^2 + \lambda(24q_1 - q_1^2 + 16q_2 - 0.5q_2^2 - 20 - 4q_1 - 4q_2 - 149)$$

$$\frac{\partial L}{\partial q_1} = 24 - 2q_1 + \lambda(24 - 2q_1 - 4) = 0$$

$$\frac{\partial L}{\partial q_2} = 16 - q_2 + \lambda(16 - q_2 - 4) = 0$$

$$\frac{\partial L}{\partial \lambda} = 20q_1 - q_1^2 + 12q_2 - 0.5q_2^2 - 169 = 0$$

$$q_1 = 11 ; q_2 = 14 ; p_1 = 13 ; p_2 = 9$$

$$R = 269 \quad ; \quad \Pi = 149$$

4) Maximisation de la fonction d'utilité de l'entreprise:

$$u = f(x_1, x_2, x_3, x_4, x_5)$$

x_1 = actionnaires

x_2 = employés

x_3 = clients

x_4 = banques

x_5 = pouvoirs publics

Dilemme: shareholders / stakeholders

Choix du niveau de production

$$\max \Pi = pq - (p_K K + p_L L)$$

$$= pf(K, L) - p_K K - p_L L$$

$$\begin{cases} \frac{\partial \Pi}{\partial K} = pf_K - p_K = 0 & pf_K = p_K \\ \frac{\partial \Pi}{\partial L} = pf_L - p_L = 0 & pf_L = p_L \end{cases}$$

Le rendement marginal en valeur de chaque facteur doit être égal au coût.

En résolvant on obtient:

$$\frac{p_K}{f_K} = \frac{p_L}{f_L} = p$$

(voir la condition de minimisation des coûts)

$$K = \varphi_1(p, p_K, p_L) ; L = \varphi_2(p, p_K, p_L)$$

Fonction d'offre:

$$q = g(p, p_K, p_L)$$

$$\text{Exemple: } q = AK^{1/2}L^{1/3}$$

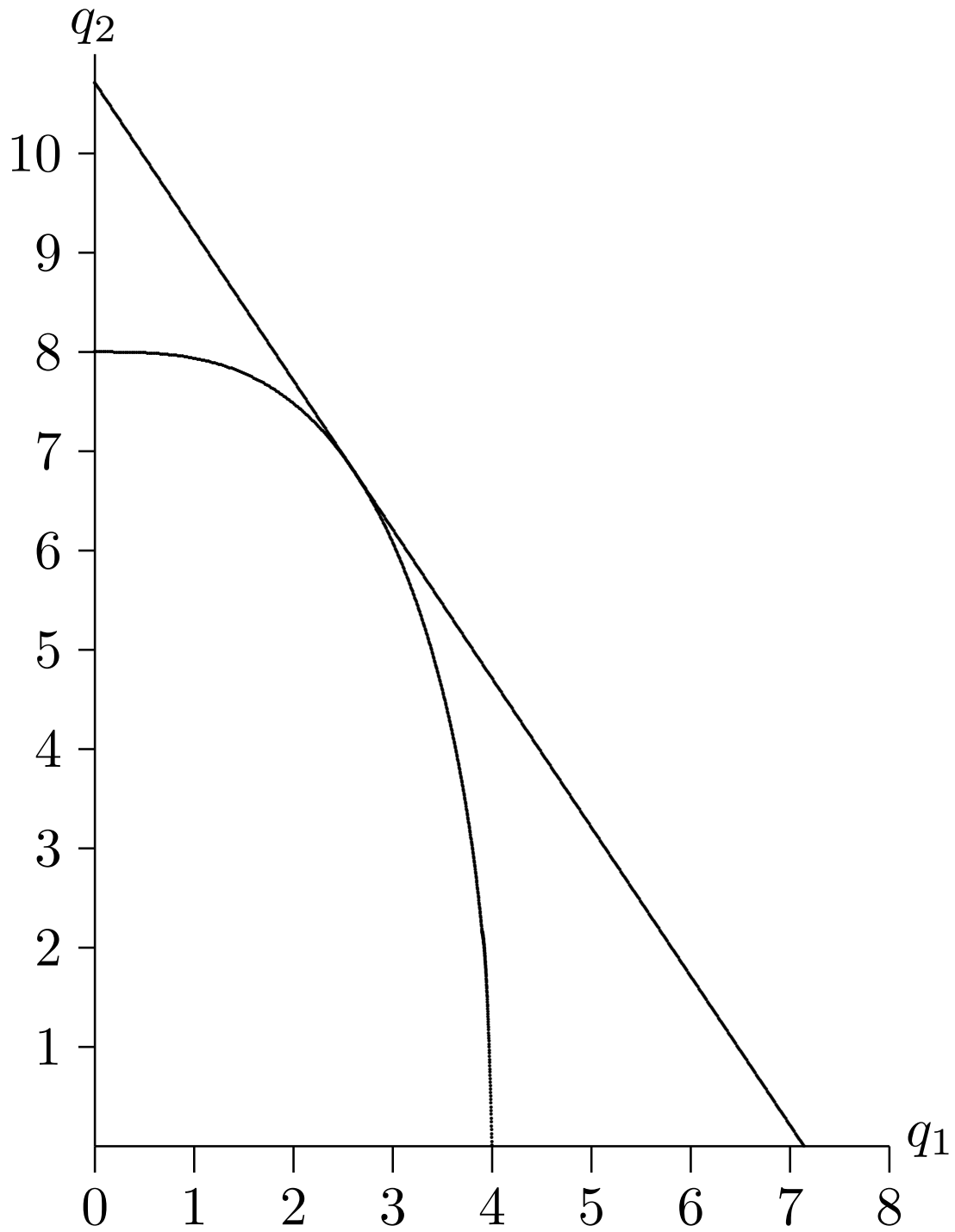
$$K = \frac{p^6 A^6}{144 p_K^4 p_L^2}$$

$$L = \frac{p^6 A^6}{216 p_K^3 p_L^3}$$

$$q = \frac{p^6 A^6}{72 p_K^3 p_L^2}$$

$$\Pi = \frac{p^6 A^6}{432 p_K^3 p_L^2}$$

Production jointe



$$q_1 = L_1^{1/3} ; q_2 = L_2^{1/2} ; L_1 + L_2 = L^o = 64$$

$$q_2 = \sqrt{L^o - q_1^3}$$

Production jointe

$$q_2 = \sqrt{L^o - q_1^3} \quad \text{ou} \quad q_2^2 + q_1^3 = L^o$$

$$\frac{dq_2}{dq_1} = \frac{-3q_1^2}{2\sqrt{L^o - q_1^3}} = \frac{-3q_1^2}{2q_2} = -TTP$$

$$\max \Pi = p_1 q_1 + p_2 q_2 \quad \text{S.C.} \quad L^o = q_1^3 + q_2^2$$

$$\mathcal{L} = p_1 q_1 + p_2 q_2 + \lambda [L^o - q_1^3 - q_2^2]$$

$$\left\{ \begin{array}{l} \frac{\partial \mathcal{L}}{\partial q_1} = p_1 - 3q_1^2 \lambda = 0 \end{array} \right. \quad (a)$$

$$\left\{ \begin{array}{l} \frac{\partial \mathcal{L}}{\partial q_2} = p_2 - 2q_2 \lambda = 0 \end{array} \right. \quad (b)$$

$$\left\{ \begin{array}{l} \frac{\partial \mathcal{L}}{\partial \lambda} = L^o - q_1^3 - q_2^2 = 0 \end{array} \right. \quad (c)$$

En prenant (a) et (b) on trouve:

$$\frac{p_1}{p_2} = \frac{3q_1^2}{2q_2} = TTP$$

Autre possibilité:

$$\Pi = p_1 q_1 + p_2 q_2 - wL^o = R_1 + R_2 - C$$

$$\left\{ \begin{array}{l} \frac{\partial \Pi}{\partial q_1} = Rm_1 - Cm_1 = 0 \quad ; \quad Rm_1 = Cm_1 \\ \frac{\partial \Pi}{\partial q_2} = Rm_2 - Cm_2 = 0 \quad ; \quad Rm_2 = Cm_2 \end{array} \right.$$